

Hawking radiation from black holes in de Sitter spaces

Qing-Quan Jiang¹

*Institute of Particle Physics, Central China Normal University, Wuhan, Hubei 430079,
People's Republic of China*

Abstract

Recently, Hawking radiation has been treated, by Robinson and Wilczek, as a compensating flux of the energy momentum tensor required to cancel gravitational anomaly at the event horizon(EH) of a Schwarzschild-type black hole. In this paper, motivated by their work, Hawking radiation from the event horizon(EH) and the de Sitter cosmological horizon(CH) of black holes in de Sitter spaces, specifically including the purely de Sitter black hole, and the static, spherically symmetric Schwarzschild-de Sitter black hole as well as the rotating Kerr-de Sitter black hole, has been studied by anomalies. The result shows that the gauge current and energy momentum tensor fluxes, required to restore gauge invariance and general coordinate covariance at the EH and the CH, are precisely equal to those of Hawking radiation from the EH and the CH, respectively. It should be noted that, gauge and gravitational anomalies taken place at the cosmological horizon(CH) arise from the fact that the effective field theory is formulated inside the CH to integrate out the classically irrelevant outgoing modes at the CH, which is different from those at the black hole horizon(EH).

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¹E-mail address: jiangqq@iopp.ccnu.edu.cn

1 Introduction

Hawking radiation from black holes, since Stephon Hawking first proved it in 1974[1], has been attracted a lot of attentions by theoretical physicists. Till now, we can derive Hawking radiation from black holes in many ways such as Euclidean quantum gravity[2], string theory [3] and a tunnelling picture [4, 5, 6] and so on. But each method exhibits a strength and weaknesses (specifically described in Ref.[7]). Recently, Robinson and Wilczek have proposed another method to derive Hawking radiation via the cancellation of gravitational anomaly at the event horizon(EH) of a Schwarzschild-type black hole[7]. In the literature, the authors attempt to formulate an effective field theory that only describes an observable physics. In this case, the global Killing vector for a static Schwarzschild-type black hole that appears locally like a time translation, but is only time-like outside the event horizon of the black hole could be a perfectly reasonable choice with which to define the energy of quantum states. However, this definition would give a divergent energy at the event horizon(EH) due to the fact that the classically irrelevant ingoing modes pile up here. To properly describe an observable physics in the effective field theory, the classically irrelevant ingoing modes must be integrated out at the event horizon(EH) to remove the divergent energy. The effective field theory thus formed no longer has observable divergences, but it now suffers from the destruction of general coordinate symmetry since the number of the ingoing modes and the outgoing modes are no longer identical. Thus the effective chiral theory contains an anomaly with respect to general coordinate symmetry, which is named as gravitational anomaly and often takes the form of nonconservation of the energy momentum tensor. To restore general coordinate covariance at the quantum level, an energy momentum tensor flux must be introduced to cancel gravitational anomaly at the event horizon(EH). The result shows that the compensating energy momentum tensor flux is exactly equal to that of $(1 + 1)$ -dimensional blackbody radiation at the Hawking temperature. In [7], a key technique, that each partial wave of quantum fields in an arbitrarily dimensional black hole behaves, by a dimensional reduction, like an independent two dimensional scalar field in the near-horizon limit, has been introduced. Subsequently, Iso etc. have extended the method to the case of a charged black hole[8] and a rotating black hole[9] via gauge and gravitational anomalies at the black hole horizon(EH). Here, the gauge charge of the effective two-dimensional fields is the electric charge e of the radiated particles for a charged black hole, but that for a rotating black hole is the radiated particles' azimuthal quantum number

m.

Till now, this method has been successfully applied in many different cases[10, 11, 12, 13, 14, 15, 16, 17], but gauge and gravitational anomalies in these observations share in common to take place at the black hole horizon(EH). In other words, present work via anomalous point of view to derive Hawking radiation is limited to study that radiated from the black hole horizon(EH). In fact, in de Sitter spaces, future infinity is spacelike, which means the universe with a repulsive Λ term will expand so rapidly that for each observer there are regions from which light can never reach him. The de Sitter cosmological horizon(CH) is the boundary of this region. Particle can be also created at such a cosmological horizon with a thermal spectrum[2]. Although it is not of much practical significance since the temperature carried by the thermal radiation is very small, researches on black holes in de Sitter spaces become important due to the following reasons: (1) The recent observed accelerating expansion of our universe indicates the cosmological constant might be a positive one[18]; (2)Conjecture about de Sitter/conformal field theory(CFT) correspondence[19].

Although the black hole horizon(EH) and the cosmological horizon(CH) share many similar properties, at which the radiation behavior behaves completely different. At the black hole horizon(EH), from which an observer who lives outside the horizon would find an outgoing flux with a thermal spectrum radiated. However, an ingoing thermal flux from the CH would be detected by an observer who lives inside the de Sitter cosmological horizon(CH). Since the cosmological horizon(CH) is a null hypersurface, the outgoing modes that fall out of the de Sitter cosmological horizon would never fall classically back, but Quantum mechanically its effect on the physics inside the cosmological horizon should be taken into account. If the effective field theory is formulated to integrate out the classically irrelevant outgoing modes at the CH, it is chiral here, but contains gauge and gravitational anomalies. To restore gauge invariance and general coordinate covariance at the quantum level, we must introduce the gauge current and energy momentum tensor fluxes to cancel these anomalies at the cosmological horizon. The result shows that these compensating fluxes are exactly equal to those of Hawking radiation from the de Sitter cosmological horizon(CH).

In this paper, taking the purely de Sitter black hole, the static, spherically symmetric Schwarzschild-de Sitter black hole and the rotating Kerr-de Sitter black hole as an example, we study Hawking radiation from the black hole horizon(EH) and the de Sitter cosmological horizon(CH) via anomalies. For the Schwarzschild-de Sitter black hole and the Kerr-de Sitter

black hole, there are a black hole horizon(EH) and a de Sitter cosmological horizon(CH) for an observer moving on the world line of constant r between the EH and the CH. Thus, the effective field theory that describes an observable physics should be formulated between the EH and the CH to, respectively, integrate out the classically irrelevant ingoing modes at the EH and the classically irrelevant outgoing modes at the CH. Now, gauge and gravitational anomalies take place at both the EH and the CH. To simplify our discussion, when dealing with Hawking radiation from the EH, we can regard that gauge and gravitational anomalies taken place in the effective field theory are due to integrate out the classically irrelevant ingoing modes at the EH, and disregard the quantum contribution of the omitted outgoing modes at the CH although they should be incorporated into the effective field theory. This means we have overlooked the effect coming from the CH when we study Hawking radiation from the EH, and assumed that the EH and the CH behave like two independent systems. In the CH case, we can similarly treat gauge and gravitational anomalies occurred in the effective field theory as arising from excluding the classically irrelevant outgoing modes at the CH, and overlook the effect of the EH. This dealing has been successfully applied in Ref.[6] to derive Hawking radiation via tunnelling from black holes in de Sitter spaces. The result shows that although gauge and gravitational anomalies take place in different ways at the black hole horizon(EH) and the cosmological horizon(CH), the gauge current and energy momentum tensor fluxes, required to restore gauge invariance and general coordinate covariance at the quantum level, are always equal to those of Hawking radiation.

The organization of the paper is as follows. In Sec.2, by extending the Robinson-Wilczek's work that Hawking radiation can be determined by anomaly cancellation at the black hole horizon(EH), we study Hawking radiation from the de Sitter cosmological horizon(CH) of the purely de Sitter black hole. Sec.3 and Sec.4 are, respectively, devoted to investigating Hawking radiation from the EH and the CH of the static, spherically symmetric Schwarzschild-de Sitter black hole and the rotating Kerr-de Sitter black hole via gauge and gravitational anomalies. Sec.5 endes up with conclusions and discussions.

2 Hawking radiation from the de Sitter black hole

In de Sitter spaces, the simplest solution for the Einstein field equation with $T_{ab} = 0$ is written as

$$ds^2 = f(r)dt^2 - \frac{1}{f(r)}dr^2 - r^2(d\theta^2 + \sin^2\theta d\phi^2), \quad (1)$$

where

$$f(r) = 1 - \frac{1}{3}\Lambda r^2. \quad (2)$$

Obviously, the cosmological horizon of the solution is located at $r_c = \sqrt{3}/\sqrt{\Lambda}$, and the surface gravity for the horizon is given by $\kappa_c = -\frac{1}{2}\partial_r f(r)|_{r=r_c} = \sqrt{\Lambda}/\sqrt{3}$. Before deriving Hawking radiation via gravitational anomaly at the horizon, we must introduce a dimensional reduction technique to reduce the scalar field theory on this metric to an effective two-dimensional theory. For the metric described by Eq.(1), the action of the scalar field reads off

$$\begin{aligned} S[\varphi] &= \frac{1}{2} \int d^4x \sqrt{-g} \varphi \nabla^2 \varphi \\ &= \frac{1}{2} \int d^4x r^2 \sin \theta \varphi \left[\frac{1}{f(r)} \partial_t^2 - \frac{1}{r^2} \partial_r r^2 f(r) \partial_r \right. \\ &\quad \left. - \frac{1}{r^2} \left(\partial_\theta^2 + \frac{1}{\sin^2 \theta} \partial_\phi^2 \right) \right] \varphi. \end{aligned} \quad (3)$$

Near the horizon, performed the partial wave decomposition of φ in terms of the spherical harmonics $\varphi = \sum_n \varphi_n(t, r) Y_n(\theta, \phi)$, the action then becomes

$$S[\varphi] = \sum_n \frac{r^2}{2} \int dt dr \varphi_n \left(\frac{1}{f(r)} \partial_t^2 - \partial_r f(r) \partial_r \right) \varphi_n. \quad (4)$$

Now each partial wave for the four dimensional de Sitter black hole can be effectively described by an infinite two-dimensional scalar field on the metric

$$ds^2 = f(r) dt^2 - \frac{1}{f(r)} dr^2, \quad (5)$$

and the dilaton background $\Psi = r^2$ whose contributions are often dropped. In the two-dimensional reduction, the energy momentum tensor is conserved due to the static background, which generates a general coordinate symmetry for the two-dimensional scalar field. When the effective field theory is formulated inside the cosmological horizon(CH) to integrate out the classically irrelevant outgoing modes at the CH, it is chiral here since an observer who lives inside the cosmological horizon no longer detect a divergent energy at the CH, but it now suffers an anomaly with respect to general coordinate symmetry, which gives a great constraint on the energy momentum tensor. For the left-handed field(ingoing modes), the consistent anomaly reads[20]

$$\nabla_\mu T_\nu^\mu = -\frac{1}{\sqrt{-g}} \partial_\mu \mathcal{N}_\nu^\mu, \quad (6)$$

where

$$\mathcal{N}_\nu^\mu = \frac{1}{96\pi} \epsilon^{\beta\mu} \partial_\alpha \Gamma_{\nu\beta}^\alpha, \quad (7)$$

and the two-dimensional Levi-Civita tensor $\epsilon^{01} = 1$. In the effective field theory, the total energy momentum tensor combines contributions from two regions $T_\nu^\mu = T_{\nu(o)}^\mu \Theta_- + T_{\nu(C)}^\mu C$, where $\Theta_- = \Theta(r_c - r - \epsilon)$ and $C = 1 - \Theta_-$ are, respectively, a scalar step function and scalar top hat function. Near the cosmological horizon $r_c - \epsilon \leq r \leq r_c$, gravitational anomaly take place in the effective field theory due to integrating out the classically irrelevant outgoing modes here, the nonconservation of the energy momentum tensor in this region satisfies $\partial_r T_{t(C)}^r = -\partial_r \mathcal{N}_t^r(r)$ (here we only consider the $\nu = t$ component since the anomaly is purely time-like), where

$$\mathcal{N}_t^r(r) = \frac{1}{192\pi} (f'^2 + f f''). \quad (8)$$

In the other region $r \leq r_c - \epsilon$, there is no anomaly, and the energy momentum tensor satisfies the conservation law as $\partial_r T_{t(o)}^r = 0$. In classical theory, the covariance of the effective action under general coordinate transformation is expressed as $\delta_\lambda W = - \int d^d x \sqrt{-g} \lambda^\nu \nabla_\mu T_\nu^\mu = 0$ where λ is the variational parameter. In our cases, the effective field theory is formulated inside the cosmological horizon to integrate out the classically irrelevant outgoing modes at the cosmological horizon(CH). Under general coordinate transformation, the effective action (without incorporating the quantum contributions of the classically irrelevant outgoing modes at the CH) changes as

$$\begin{aligned} -\delta_\lambda W &= \int dt dr \sqrt{-g} \lambda^t \nabla_\mu (T_{t(o)}^\mu \Theta_- + T_{t(C)}^\mu C) \\ &= \int dt dr \lambda^t [(T_{t(C)}^r - T_{t(o)}^r + \mathcal{N}_t^r) \delta(r - r_c + \epsilon) \\ &\quad - \partial_r (C \mathcal{N}_t^r)]. \end{aligned} \quad (9)$$

In Eq.(9), the the second term should be cancelled by the quantum effect of the classically irrelevant outgoing modes at the CH, whose contributions to the total energy momentum tensor are $C \mathcal{N}_t^r$. To restore general coordinate covariance at the quantum level, the coefficient of the delta function should also vanish, which means

$$a_o = a_c + \mathcal{N}_t^r(r_c), \quad (10)$$

where $a_o = T_{t(o)}^r$ is the energy flow observed by an observer who lives inside the cosmological horizon(CH), and

$$a_c = T_{t(C)}^r + \int_{r_c}^r dr \partial_r \mathcal{N}_t^r, \quad (11)$$

are the value of the energy flow at the cosmological horizon(CH). To ensure the regularity of the physics quantities at the CH, the covariant energy momentum tensor, which is related to the consistent one by [8]

$$\tilde{T}_{t(C)}^r = T_{t(C)}^r - \frac{1}{192\pi}(ff'' - 2f'^2), \quad (12)$$

should vanish here. This regular condition determines the energy momentum tensor flux at the CH should be taken the form as

$$a_c = -\frac{\kappa_c^2}{24\pi} = -2\mathcal{N}_t^r(r_c). \quad (13)$$

Thus, to cancel gravitational anomaly at the cosmological horizon(CH), the total energy momentum tensor flux is given by

$$a_o = -\mathcal{N}_t^r(r_c) = -\frac{\pi}{12}T_c^2, \quad (14)$$

where

$$T_c = \frac{\kappa_c}{2\pi} = \frac{\sqrt{\lambda}}{2\sqrt{3}\pi}, \quad (15)$$

is the Hawking temperature at the cosmological horizon of the black hole. In (14), the negative sign denotes that the effective field theory would absorb the energy momentum tensor flux to ensure general coordinate covariance at the quantum level. In fact, this absorbing energy momentum tensor flux is exactly equal to that of Hawking radiation from the CH. For fermions, the Hawking distribution at the de Sitter cosmological horizon(CH) of the black hole is given by $\mathcal{N}(\omega) = -1/[\exp(\frac{\omega}{T_c}) + 1]$ (here the negative sign denotes blackbody radiation is radiated into the black hole from the CH). With this distribution, the energy momentum tensor flux is given by

$$F_c = \int_0^\infty \frac{\omega}{\pi} \mathcal{N}(\omega) d\omega = -\frac{\pi}{12}T_c^2. \quad (16)$$

Obviously, the total energy momentum tensor flux, required to cancel gravitational anomaly at the cosmological horizon(CH) and restore general coordinate covariance at the quantum level, has an equivalent form as that of Hawking radiation from the CH.

In the following section, we will further extend the Robinson-Wilczek's method to study Hawking radiation from the event horizon(EH) and the de Sitter cosmological horizon(CH) of the Schwarzschild-de Sitter black hole. In the two-dimensional reduction, if the effective field theory is formulated between the EH and the CH to, respectively, exclude the classically

irrelevant ingoing modes at the EH and the classically irrelevant outgoing modes at the CH, gravitational anomaly would take place at both the EH and the CH. In our discussion, we take the simplest case as discussed above to derive Hawking fluxes from black holes in de Sitter spaces.

3 Hawking radiation from the Schwarzschild-de Sitter black hole

The Schwarzschild solution with a repulsive constant Λ represents a black hole in asymptotically de Sitter space. The metric of a four-dimensional Schwarzschild-de Sitter black hole can be written as

$$ds^2 = f(r)dt^2 - \frac{1}{f(r)}dr^2 - r^2(d\theta^2 + \sin^2\theta d\phi^2), \quad (17)$$

where

$$f(r) = 1 - \frac{2M}{r} - \frac{1}{3}\Lambda r^2. \quad (18)$$

If $\Lambda > 0$ and $9\Lambda M^2 < 1$, $f(r)$ is zero at the two positive values of r . In which, the smaller positive one, denoted by r_h can be regarded as the position of the black hole horizon, while the larger value r_c represents the position of the de Sitter cosmological horizon, and r_- is the negative root of $f(r) = 0$. The surface gravities on the black hole horizon(EH) and the cosmological horizon(CH) read as

$$\begin{aligned} \kappa_h &= \frac{1}{2}\partial_r f(r)|_{r=r_h} = \frac{\Lambda}{6r_h}(r_c - r_h)(r_h - r_-), \\ \kappa_c &= -\frac{1}{2}\partial_r f(r)|_{r=r_c} = \frac{\Lambda}{6r_c}(r_c - r_h)(r_c - r_-), \end{aligned} \quad (19)$$

respectively. After a dimensional reduction technique near the EH or the CH, the scalar field theory in the original dimensions can always be treated as an infinite collection of $(1+1)$ -dimensional fields on the metric described by Eq.(5) only replacing $f(r)$ with that in Eq.(18). In this section, Hawking radiation from both the EH and the CH will be derived from anomalous point of view. Now we first study the energy momentum tensor flux and gravitational anomaly at the EH.

3.1 Hawking radiation from the EH

In the Schwarzschild-de Sitter spacetime, the surface $r = r_h$ and $r = r_c$ are, respectively, the black hole horizon(EH) and the de Sitter cosmological horizon(CH) for an observer

moving on the word lines of constant r between the EH and the CH. In the two-dimensional reduction, the effective field theory that only describes an observable physics should be then formulated between the event horizon(EH) and the de Sitter cosmological horizon(CH) to, respectively, integrate out the ingoing modes at the EH and the outgoing modes at the CH. Now, gravitational anomaly takes place at both the EH and the CH. When dealing with Hawking radiation from the EH, we assume gravitational anomaly taken place in the effective field theory is due to exclude the classically irrelevant ingoing modes at the EH, and disregard the quantum contribution of the omitted outgoing modes at the CH although they should be incorporated into the effective field theory. Near the EH $r_h \leq r \leq r_h + \epsilon$, the nonconservation of the energy momentum tensor then satisfies the anomalous equation as

$$\nabla_\mu T_{\nu(H)}^\mu = \frac{1}{\sqrt{-g}} \partial_\mu \mathcal{N}_\nu^\mu, \quad (20)$$

for right-handed fields(outgoing modes), where \mathcal{N}_ν^μ has a completely same form as Eq.(7). In the other region $r_h + \epsilon \leq r \leq r_c$, without any anomalies, the energy momentum tensor satisfies the conservation equation as $\nabla_\mu T_{\nu(o)}^\mu = 0$. As the anomaly is purely time-like, the anomalous energy momentum tensor near the EH satisfies $\partial_r T_{t(H)}^r = \partial_r \mathcal{N}_t^r$, where \mathcal{N}_t^r is given by Eq.(8) only replacing $f(r)$ with that in Eq.(18), and the conservation energy momentum tensor in the other region satisfies $\partial_r T_{t(o)}^r = 0$. Under general coordinate transformation, the effective action changes as

$$\begin{aligned} -\delta_\lambda W &= \int dt dr \sqrt{-g} \lambda^t \nabla_\mu (T_{t(o)}^\mu \Theta_+ + T_{t(H)}^\mu H) \\ &= \int dt dr \lambda^t \left[(T_{t(o)}^r - T_{t(H)}^r + \mathcal{N}_t^r) \delta(r - r_h - \epsilon) \right. \\ &\quad \left. + \partial_r (H \mathcal{N}_t^r) \right], \end{aligned} \quad (21)$$

where $\Theta_+ = \Theta(r - r_h - \epsilon)$, and $H = 1 - \Theta_+$ are, respectively, scalar step function and top hat function. In (21), the classically irrelevant ingoing modes at the EH have been integrated out, so the second term is cancelled by its quantum effect, whose contributions to the total energy momentum tensor flux is $-H \mathcal{N}_t^r$. To restore general coordinate covariance at the quantum level, the coefficient of the delta function should be also vanished. This relates

$$d_o = d_h - \mathcal{N}_t^r(r_h), \quad (22)$$

where $d_o = T_{t(o)}^r$ is the energy flow observed by an observer who lives between the EH and the CH, and

$$d_h = T_{t(H)}^r - \int_{r_h}^r dr \partial_r \mathcal{N}_t^r, \quad (23)$$

is the energy flow at the EH. To determine the total energy momentum tensor flux, we impose the covariant energy momentum tensor is vanished at the EH, which corresponds to the regularity condition of the physical quantities. Since the covariant energy momentum tensor is written in the consistent one by[8]

$$\tilde{T}_{t(H)}^r = T_{t(H)}^r + \frac{1}{192\pi} [ff'' - 2(f')^2], \quad (24)$$

the vanishing condition determines the energy momentum tensor flux at the EH

$$d_h = \frac{\kappa_h^2}{24\pi} = 2\mathcal{N}_t^r(r_h). \quad (25)$$

Thus, the compensating energy momentum tensor flux, required to cancel gravitational anomaly and restore general coordinate covariance at the quantum level, is given by

$$d_o = \mathcal{N}_t^r(r_h) = \frac{\pi}{12} T_h^2, \quad (26)$$

where $T_h = \kappa_h/(2\pi)$ is the Hawking temperature at the event horizon(EH) of the black hole. At the EH, as the Planckian distribution of blackbody radiation moving in the positive r direction at the Hawking temperature T_h takes the form as $\mathcal{N}(\omega) = 1/[\exp(\frac{\omega}{T_h}) + 1]$ for fermions, the energy momentum tensor flux with this distribution is calculated as

$$F_h = \int_0^\infty \frac{\omega}{\pi} \mathcal{N}(\omega) d\omega = \frac{\pi}{12} T_h^2. \quad (27)$$

Compared the energy momentum tensor flux derived from the cancellation condition of gravitational anomaly at the EH with that from Hawking radiation with the Planckian distribution at the Hawking temperature T_h shows that Hawking radiation is capable of restoring general coordinate covariance at the quantum level.

3.2 Hawking radiation from the CH

The previous subsection tells us that the compensating energy momentum tensor flux at the EH, required to cancel gravitational anomaly and restore general coordinate covariance at the quantum level, is exactly equal to that of $(1+1)$ -dimensional blackbody radiation at the Hawking temperature. The simplest case, that gravitational anomaly in the effective field theory only occurs at the EH and the quantum contributions of the omitted outgoing modes at the CH is out of consideration, has been adopted in our discussion. In this subsection, we will concentrate on studying Hawking radiation from the cosmological horizon(CH) of the black hole via anomalous point of view. Similarly, we can think that gravitational anomaly

taken place in the effective field theory is due to exclude the classically irrelevant outgoing modes at the CH, and the quantum effect of the omitted ingoing modes at the EH is out of consideration. Thus, the energy momentum tensor in the effective field theory is contributed as $T_\nu^\mu = T_{\nu(o)}^\mu \Theta_- + T_{\nu(C)}^\mu C$, where $\Theta_- = \Theta(r_c - r - \epsilon)$ is a scalar step function, and $C = 1 - \Theta_-$ denotes a scalar top hat function. According to Sec.2, gravitational anomaly for left-handed fields(ingoing modes) gives a great constraint on the consistent energy momentum tensor as described by Eq.(6). Under general coordinate transformation, the variation of the effective action is given by

$$\begin{aligned} -\delta W &= \int dt dr \sqrt{-g} \lambda^t \nabla_\mu (T_{t(o)}^\mu \Theta_- + T_{t(C)}^\mu C) \\ &= \int dt dr \lambda^t [(T_{t(C)}^r - T_{t(o)}^r + \mathcal{N}_t^r) \delta(r - r_c + \epsilon) \\ &\quad - \partial_r(C \mathcal{N}_t^r)]. \end{aligned} \quad (28)$$

In Eq.(28), the classically irrelevant outgoing modes at the CH has been integrated out. As the original underlying theory is, of course, covariant and the last term should be cancelled by the quantum effect of the classically irrelevant outgoing modes at the CH, whose contributions to the total energy momentum tensor are $C \mathcal{N}_t^r$. To restore diffeomorphism covariance at the quantum level, the coefficient of the delta function should also vanish, which relates

$$f_o = f_c + \mathcal{N}_t^r(r_c), \quad (29)$$

where $f_o = T_{t(o)}^r$ is the flux of the energy momentum tensor observed by an observer who lives between the EH and the CH, and

$$f_c = T_{t(C)}^r + \int_{r_c}^r dr \partial_r \mathcal{N}_t^r, \quad (30)$$

is the energy flow at the CH. The covariant energy momentum tensor, which is related to the consistent one as Eq.(12), should be vanished to assure regularity of the physical quantities at the CH. That condition determines the flux of the energy momentum tensor at the CH as

$$f_c = -\frac{\kappa_c^2}{24\pi} = -2\mathcal{N}_t^r(r_c), \quad (31)$$

where κ_c is the surface gravity at the de Sitter cosmological horizon(CH) of the black hole. Now the total flux of the energy momentum tensor, required to cancel gravitational anomaly at the CH, is written as

$$f_o = -\mathcal{N}_t^r(r_c) = -\frac{\pi}{12} T_c^2, \quad (32)$$

where $T_c = \kappa_c/(2\pi)$ is the Hawking temperature at the CH of the black hole, and the negative sign denotes the effective field theory must absorb the energy flow to cancel gravitational anomaly at the CH. In fact, the energy momentum tensor flux of Hawking radiation at the CH of the black hole has a same form as that derived from the vanishing condition of gravitational anomaly at the CH. For fermions, the Hawking distribution at the CH of the black hole should be taken the form as $\mathcal{N}(\omega) = -1/[\exp(\frac{\omega}{T_c}) + 1]$ (the negative sign denotes blackbody radiation at the CH of the black hole is moving in the negative r direction), where T_c is the Hawking temperature at the CH. Integrating the Hawking distribution, we obtain the flux of the energy momentum tensor

$$F_c = \int_0^\infty \frac{\omega}{\pi} \mathcal{N}(\omega) d\omega = -\frac{\pi}{12} T_c^2. \quad (33)$$

Obviously, the total energy momentum tensor flux required to cancel gravitational anomaly at the CH of the black hole is precisely equal to that of Hawking radiation from the CH.

In a word, to restore general coordinate covariance at the quantum level, each partial wave of the scalar field at the EH and the CH must be, respectively, in a state with the energy momentum tensor flux given by Eq.(26) and Eq.(32). In addition, these energy momentum tensor fluxes are exactly equal to those of $(1+1)$ -dimensional blackbody radiation .

The following section is devoted to investigating Hawking radiation from the EH and the CH of the rotating Kerr-de Sitter black hole via gauge and gravitational anomalies. In this background, there are also the black hole horizon(EH) and the de Sitter cosmological horizon(CH) for an observer who lives between the EH and the CH. After a dimensional reduction technique, the effective two-dimensional theory for each partial wave exhibits an $U(1)$ gauge symmetry, which originates from the isometry of the black hole along ϕ direction, and a general coordinate symmetry. When the effective field theory is formulated between the EH and the CH to, respectively, integrate out the classically irrelevant ingoing modes at the EH and the classically irrelevant outgoing modes at the CH, $U(1)$ gauge and gravitational anomalies take place. It is expected that the total $U(1)$ gauge current and energy momentum tensor fluxes, which are required to cancel these anomalies, are precisely equal to those of Hawking radiation.

4 Hawking radiation from the Kerr-de Sitter black hole

The Kerr-de Sitter black hole can be expressed in Boyer-Lindquist coordinates as[21]

$$\begin{aligned} ds^2 = & \frac{\Delta}{\rho^2} \left(dt - \frac{a}{\Xi} \sin^2 \theta d\phi \right)^2 - \rho^2 \left(\frac{dr^2}{\Delta} + \frac{d\theta^2}{\Delta_\theta} \right) \\ & - \frac{\Delta_\theta \sin^2 \theta}{\rho^2} \left(adt - \frac{r^2 + a^2}{\Xi} d\phi \right)^2, \end{aligned} \quad (34)$$

where

$$\begin{aligned} \rho^2 &= r^2 + a^2 \cos^2 \theta, \quad \Xi = 1 + \frac{a^2}{l^2}, \\ \Delta &= (r^2 + a^2) \left(1 - \frac{r^2}{l^2} \right) - 2mr, \\ \Delta_\theta &= 1 + \frac{a^2}{l^2} \cos^2 \theta, \quad \frac{1}{l^2} = \frac{\Lambda}{3}. \end{aligned} \quad (35)$$

Here m and a are the mass and rotational parameters, respectively. Λ is the cosmological constant parameter. The event horizon(EH) (r_+) and the de Sitter cosmological horizon(CH) (r_c) are given by the equation $\Delta = 0$. To derive Hawking radiation from the black hole via anomalous point of view, we must first introduce a dimensional reduction technique to reduce the higher-dimensional theory to the effective two-dimensional one. Ref.[16] tells us, performing the partial wave decomposition of the scalar field in terms of the spherical harmonics as $\varphi = \sum_{l,m} \varphi_{lm}(t, r) Y_{lm}(\theta, \phi)$ and transforming to the tortoise coordinate defined by $dr_*/dr = (r^2 + a^2)/\Delta \equiv 1/f(r)$, one can easily observe, near the EH or the CH, each partial wave of the scalar field φ in the four-dimensional Kerr-de Sitter black hole can be effectively described by an infinite collection complex scalar field in the background of a $(1+1)$ -dimensional metric, the dilaton Ψ , and the $U(1)$ gauge field \mathcal{A}_μ as

$$\begin{aligned} g_{tt} &= f(r) = \frac{\Delta}{r^2 + a^2}, \quad g_{rr} = -\frac{1}{f(r)}, \\ \Psi &= \frac{r^2 + a^2}{\Xi}, \quad \mathcal{A}_t = -\frac{\Xi a}{r^2 + a^2}, \quad \mathcal{A}_r = 0, \end{aligned} \quad (36)$$

where the gauge charge for the $U(1)$ gauge field is the azimuthal quantum number m . The specific dimensional reduction procedure can be referred to Ref.[16]. Now the physics near the horizon of the black hole can be effectively described by the effective two-dimensional theory.

In the Kerr-de Sitter spacetime, there are two event horizons, namely the black hole horizon(EH) and the de Sitter cosmological horizon(CH), for an observer who lives between

the EH and the CH. The $U(1)$ gauge and gravitational anomalies take place in the effective field theory due to, respectively, integrate out the classically irrelevant ingoing modes at the EH and the classically irrelevant outgoing modes at the CH. In the following subsections, Hawking radiation from the EH and the CH will be discussed via gauge and gravitational anomalies. To simplify our discussion, when dealing with Hawking radiation from the EH, we think gauge and gravitational anomalies only take place at the EH due to exclude the classically irrelevant ingoing modes, and disregard the quantum contributions of the omitted outgoing modes at the CH. That is to say, we treat the EH and the CH as two independent physical systems with their probably interactions being overlooked.

4.1 Hawking radiation from the EH

As before, the first thing is to study the $U(1)$ gauge current flux and the $U(1)$ gauge anomaly at the EH. When the effective field theory is formulated to integrate out the classically irrelevant ingoing modes at the EH, but ignore the quantum contributions of the omitted outgoing modes at the CH, the $U(1)$ gauge current becomes anomalous at the EH. Near the black hole horizon(EH) $r_+ \leq r \leq r_+ + \epsilon$, it satisfies the anomalous equation as $\partial_r \mathcal{J}_{(H)}^r = m^2 \partial_r \mathcal{A}_t / (4\pi)$. In the other region $r_+ + \epsilon \leq r \leq r_c$, without any anomalies, the $U(1)$ gauge current obeys the conservation equation $\partial_r \mathcal{J}_{(o)}^r = 0$. Under gauge transformation, the effective action changes as

$$\begin{aligned} -\delta W &= \int dt dr \lambda \nabla_\mu (\mathcal{J}_{(H)}^\mu H + \mathcal{J}_{(o)}^\mu \Theta_+) \\ &= \int dt dr \lambda \left[(\mathcal{J}_{(o)}^r - \mathcal{J}_{(H)}^r + \frac{m^2}{4\pi} \mathcal{A}_t) \delta(r - r_+ - \epsilon) \right. \\ &\quad \left. + \partial_r \left(\frac{m^2}{4\pi} \mathcal{A}_t H \right) \right]. \end{aligned} \quad (37)$$

Here, $\Theta_+(r) = \Theta(r - r_+ - \epsilon)$ and $H(r) = 1 - \Theta_+(r)$ are the scalar step and top hat functions, respectively. In Eq.(37), the second term would be cancelled by the quantum effect of the classically irrelevant ingoing modes at the EH, whose contributions to the total gauge current are $-m^2 \mathcal{A}_t H / (4\pi)$. To demand the full quantum theory gauge invariance at the quantum level, the coefficient of the delta functions should be nullified, which relates

$$g_o = g_h - \frac{m^2}{4\pi} \mathcal{A}_t(r_+). \quad (38)$$

In (38), $g_o = \mathcal{J}_{(o)}^r$ is the gauge current flux observed by an observer who lives between the EH and the CH, and

$$g_h = \mathcal{J}_{(H)}^r - \frac{m^2}{4\pi} \int_{r_+}^r dr \partial_r \mathcal{A}_t, \quad (39)$$

is the value of the gauge current at the EH. In order to fix the total $U(1)$ gauge current flux, we impose a constraint that the covariant gauge current related to the consistent one by $\tilde{\mathcal{J}}_{(H)}^r = \mathcal{J}_{(H)}^r + m^2 \mathcal{A}_t H / (4\pi)$ is vanished at the EH, that condition reads

$$g_o = -\frac{m^2}{2\pi} \mathcal{A}_t(r_+) = \frac{\Xi m^2 a}{2\pi(r_+^2 + a^2)}. \quad (40)$$

This is the compensating gauge current flux to cancel the $U(1)$ gauge anomaly at the EH, which in fact, corresponds to the angular momentum flow of Hawking radiation from the EH of the black hole.

Next, we focus on studying the energy momentum tensor flux radiated from the EH. When excluding the classically irrelevant ingoing modes at the EH, the total energy momentum tensor for the effective field theory contains two contributions, that is $T_\nu^\mu = T_{\nu(o)}^\mu \Theta_+(r) + T_{\nu(H)}^\mu H(r)$. Here, $T_{\nu(o)}^\mu$ satisfies the conservation law of the energy momentum tensor in the gauge field background $\partial_r T_{t(o)}^r = g_o \partial_r \mathcal{A}_t$, while the other component obeys the anomalous equation[8]

$$\partial_r T_{t(H)}^r = \mathcal{J}_{(H)}^r \partial_r \mathcal{A}_t + \mathcal{A}_t \partial_r \mathcal{J}_{(H)}^r + \partial_r \mathcal{N}_t^r, \quad (41)$$

where \mathcal{N}_t^r takes the same form as Eq.(8) only replacing $f(r)$ with that in Eq.(36). Under general coordinate transformations, the effective action changes as

$$\begin{aligned} -\delta W &= \int dt dr \lambda^t \left[g_o \partial_r \mathcal{A}_t + \partial_r \left(\frac{m^2}{4\pi} \mathcal{A}_t^2 + \mathcal{N}_t^r \right) H \right. \\ &\quad \left. + \left(T_{t(o)}^r - T_{t(H)}^r + \frac{m^2}{4\pi} \mathcal{A}_t^2 + \mathcal{N}_t^r \right) \delta(r - r_+ - \epsilon) \right]. \end{aligned} \quad (42)$$

In (42), the first item is the classical effect of the background gauge field for constant current flow, and the second term is cancelled by the quantum effect of the classically irrelevant ingoing modes at the EH, whose contributions to the total energy momentum tensor are $-(m^2 \mathcal{A}_t^2 / (4\pi) + \mathcal{N}_t^r) H$. To demand the effective action general coordinate covariance at the quantum level, the coefficient of the delta function should also vanish at the EH, which relates

$$k_o = k_h + \frac{m^2}{4\pi} \mathcal{A}_t^2(r_+) - \mathcal{N}_t^r(r_+), \quad (43)$$

where

$$\begin{aligned} k_o &= T_{t(o)}^r - g_o \mathcal{A}_t(r), \\ k_h &= T_{t(H)}^r - \int_{r_+}^r dr \partial_r \left[g_o \mathcal{A}_t + \frac{m^2}{4\pi} \mathcal{A}_t^2 + \mathcal{N}_t^r \right], \end{aligned} \quad (44)$$

are, respectively, the energy flow observed by an observer who lives between the EH and the CH, and that at the EH. Taking the form of the covariant energy momentum tensor as Eq.(24), and further imposing the vanishing condition on it, the total flux of the energy momentum tensor is given by

$$k_o = \frac{\Xi^2 m^2 a^2}{4\pi(r_+^2 + a^2)^2} + \frac{\pi}{12} T_+^2, \quad (45)$$

where $T_+ = \partial_r f(r)/(4\pi)|_{r=r_+}$ is the Hawking temperature at the EH of the black hole. In fact, the $U(1)$ gauge current and energy momentum tensor fluxes, derived from the vanishing condition of the $U(1)$ gauge and gravitational anomalies at the EH, are exactly equal to those of Hawking radiation with the Planckian distribution $\mathcal{N}_{\pm m}(\omega) = 1/[\exp(\frac{\omega \pm m \mathcal{A}_t(r_+)}{T_+}) + 1]$. So Hawking radiation from the EH of the black hole can be effectively determined by anomalous point of view.

Different from the radiation behavior at the EH, Hawking radiation is radiated into the black hole from the CH. In anomalous point of view, the formulated effective theory must absorb the gauge current and energy momentum tensor fluxes to cancel gauge and gravitational anomalies at the CH. We expect that these absorbing fluxes are precisely equal to those of Hawking radiation. Here, we take the simplest case that gauge and gravitational anomalies only take place at the CH due to excluding the classically irrelevant outgoing modes, and disregard the quantum contributions of the omitted ingoing modes at the EH.

4.2 Hawking radiation from the CH

Now we first determine the $U(1)$ gauge current flux and the $U(1)$ gauge anomaly at the CH. Similarly, as the $U(1)$ gauge anomaly takes place at the CH due to integrating out the classically irrelevant outgoing modes, the $U(1)$ gauge current becomes anomalous near the CH $r_c - \epsilon \leq r \leq r_c$, and obeys the anomalous equation $\partial_r \mathcal{J}_{(C)}^r = -m^2 \partial_r \mathcal{A}_t/(4\pi)$ for the left-handed fields(ingoing modes). In the other region, the current is conserved $\partial_r \mathcal{J}_{(o)}^r = 0$. The total $U(1)$ gauge current can be written as a sum of the two regions, that is $\mathcal{J}^r = \mathcal{J}_{(o)}^r \Theta_- + \mathcal{J}_{(C)}^r C$. Here, $\Theta_- = \Theta(r_c - r - \epsilon)$ and $C = 1 - \Theta_-$ are, respectively, the

scalar step function and top hat function. Under gauge transformation, the variance of the effective action (without the classically irrelevant outgoing modes at the CH) can be written as

$$\begin{aligned} -\delta W &= \int dt dr \lambda \left[-\partial_r \left(\frac{m^2}{4\pi} \mathcal{A}_t C \right) \right. \\ &\quad \left. + \left(\mathcal{J}_{(C)}^r - \mathcal{J}_{(o)}^r + \frac{m^2}{4\pi} \mathcal{A}_t \right) \delta(r - r_c + \epsilon) \right]. \end{aligned} \quad (46)$$

In (46), the first term is cancelled by the quantum effect of the classically irrelevant outgoing modes at the CH, whose contributions to the total current are $m^2 \mathcal{A}_t C / (4\pi)$. The effective action should be gauge invariance at the quantum level, which relates

$$h_o = h_c + \frac{m^2}{4\pi} \mathcal{A}_t(r_c), \quad (47)$$

where h_o is the $U(1)$ gauge current flux observed by an observer who lives between the EH and the CH, and h_c is that at the CH. Its values can be easily determined as

$$\begin{aligned} h_o &= \mathcal{J}_{(o)}^r, \\ h_c &= \mathcal{J}_{(C)}^r + \frac{m^2}{4\pi} \int_{r_c}^r dr \partial_r \mathcal{A}_t. \end{aligned} \quad (48)$$

Imposing the covariant current, which is related to the consistent one as $\tilde{\mathcal{J}}^r = \mathcal{J}^r - m^2 \mathcal{A}_t C / (4\pi)$, vanishes at the CH, we can easily determine the value of the $U(1)$ gauge current flux to be

$$h_o = \frac{m^2}{2\pi} \mathcal{A}_t(r_c) = -\frac{\Xi m^2 a}{2\pi(r_c^2 + a^2)}. \quad (49)$$

This gauge current flux corresponds to the angular momentum flux of Hawking radiation from the CH of the black hole. The negative sign denotes that the effective field theory must absorb the gauge current flux to ensure gauge invariance at the quantum level.

Also, we can determine the total energy momentum tensor flux radiated from the CH of the Kerr-de Sitter black hole. In addition to gauge symmetry, the effective two-dimensional theory for each partial wave has general coordinate symmetry. Here we take the simplest case discussed above. Near the CH $r_c - \epsilon \leq r \leq r_c$, the effective field theory contains $U(1)$ gauge and gravitational anomalies, which gives a great constraint on the energy momentum tensor

$$\partial_r T_{t(C)}^r = \mathcal{J}_{(C)}^r \partial_r \mathcal{A}_t + \mathcal{A}_t \partial_r \mathcal{J}_{(C)}^r - \partial_r \mathcal{N}_t^r. \quad (50)$$

In the other region, since there is a $U(1)$ gauge field background, the energy momentum tensor satisfies the modified conservation equation $\partial_r T_{t(o)}^r = h_o \partial_r \mathcal{A}_t$. Under general coordinate transformation, the effective action (here we omit the contributions of the classically

irrelevant outgoing modes at the CH) changes as

$$\begin{aligned} -\delta W &= \int dt dr \lambda^t \left[h_o \partial_r \mathcal{A}_t - \partial_r \left(\frac{m^2}{4\pi} \mathcal{A}_t^2 + \mathcal{N}_t^r \right) C \right. \\ &\quad \left. + \left(T_{t(C)}^r - T_{t(o)}^r + \frac{m^2}{4\pi} \mathcal{A}_t^2 + \mathcal{N}_t^r \right) \delta(r - r_c + \epsilon) \right]. \end{aligned} \quad (51)$$

In (51), the first term is the classical effect of the background gauge field for constant current flow. The second term is cancelled by the quantum effect of the classically irrelevant outgoing modes at the CH, whose contributions to the total energy momentum tensor are $(m^2 \mathcal{A}_t^2 / (4\pi) + \mathcal{N}_t^r)C$. To restore general coordinate covariance at the quantum level, the coefficient of the delta function should also vanish. So we have

$$n_o = n_c - \frac{m^2}{4\pi} \mathcal{A}_t(r_c) + \mathcal{N}_t^r(r_c), \quad (52)$$

where

$$\begin{aligned} n_o &= T_{t(o)}^r - h_o \mathcal{A}_t(r), \\ n_c &= T_{t(C)}^r - \int_{r_c}^r dr \partial_r \left[h_o \mathcal{A}_t - \frac{m^2}{4\pi} \mathcal{A}_t^2 - \mathcal{N}_t^r \right], \end{aligned} \quad (53)$$

are the energy flow observed by an observer who lives between the EH and the CH, and that at the CH, respectively. To assure the regularity of the physical quantities, we impose the covariant energy momentum tensor, which is related to the consistent one as described by Eq.(12), vanishes at the CH, that condition determines the total energy momentum tensor flux as

$$n_o = -\frac{\Xi^2 m^2 a^2}{4\pi(r_c^2 + a^2)^2} - \frac{\pi}{12} T_c^2. \quad (54)$$

In (54), $T_c = -\partial_r f(r)/(4\pi)|_{r=r_c}$ is the Hawking temperature at the CH of the black hole. The negative sign denotes that the energy momentum tensor flux is radiated into the effective theory to ensure general coordinate covariance at the quantum level.

In fact, these absorbing gauge current and energy momentum tensor fluxes required to restore gauge invariance and general coordinate covariance at the the quantum level, and respectively expressed by Eq.(49) and Eq.(54), are exactly equal to those of blackbody radiation moving in the negative r direction with the Hawking distribution at the CH. For fermions, the Hawking distribution at the CH formally takes the form as $\mathcal{N}_{\pm m}(\omega) = -1/[\exp(\frac{\omega \pm m A_t(r_c)}{T_c}) + 1]$ (here the negative sign denotes Hawking radiation is radiated into the black hole from the CH). Integrating this distribution, the angular momentum and energy momentum tensor fluxes at the CH can be obtained to take the same form as Eq.(49) and Eq.(54), respectively.

5 Conclusions and Discussions

In this paper, we study Hawking radiation of different-type black holes in de Sitter spaces via anomalous point of view, specifically including that of the purely de Sitter black hole, and the static, spherically symmetric Schwarzschild-de Sitter black hole as well as the rotating Kerr-de Sitter black hole. These black holes shares in common to have a de Sitter cosmological horizon(CH). At the CH, the outgoing modes that fall out of the de Sitter cosmological horizon(CH) would never fall classically back since the cosmological horizon(CH) is a null hypersurface, but Quantum mechanically its contributions on the physics inside the cosmological horizon(CH) should be taken into account. In the two-dimensional reduction, when the effective field theory is formulated to exclude the classically irrelevant outgoing modes at the CH, it is chiral here, but contains gauge or gravitational anomaly. In order to cancel these anomalies and restore gauge invariance or general coordinate covariance at the quantum level, each partial wave of the scalar field must be in a state with a net gauge current flux given by Eq.(49) and energy momentum tensor flux given by Eq.(14), Eq.(32) and Eq.(54). The result shows that these absorbing fluxes are exactly equal to those of Hawking radiation from the de Sitter cosmological horizon(CH).

In the case of black holes with a repulsive Λ term, there are an event horizon(EH) and a de Sitter cosmological horizon(CH) for an observer who moves on the world line of constant r between the EH and the CH. The effective field theory that describes an observable physics should then be formulated between the EH and the CH to, respectively, integrate out the classically irrelevant ingoing modes at the EH and the classically irrelevant outgoing modes at the CH. To simplify our discussion, we have treated the EH and the CH as two independent systems. That is to say, when dealing with Hawking radiation from the EH, we have overlooked the effect coming from the CH. Similarly, the effect coming from the EH is ignored when we consider the de Sitter radiation from the CH. This simplification can be seen in Ref.[6] to derive Hawking radiation via tunnelling from black holes in de Sitter spaces. The result shows that Hawking fluxes from the black hole horizon(EH) and the de Sitter cosmological horizon(CH) are capable of cancelling gauge or gravitational anomaly and restoring gauge invariance or general coordinate covariance at the quantum level.

In fact, the Robinson-Wilczek's method can be universally extended to derive Hawking fluxes from any static or stationary black hole, which takes the form as $ds^2 = f(r)dt^2 - g^{-1}(r)dr^2$ after a dimensional reduction technique near the horizon. In our discussion, $f(r) =$

$g(r)$ and the determinant of its diagonal metric is the unity. If its value differs from the unity, namely $f(r) \neq g(r)$, the Robinson-Wilczek's derivation of Hawking radiation via anomalies can also be applicable, but some formula need to be modified by $\sqrt{f(r)/g(r)}$ (specifically see in Ref.[15]).

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